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Optimal $(n-1)$-reliable design of distributed energy supply systems

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Distributed energy supply systems are most efficiently designed by mathematical optimization. However, optimization models often assume availability of all components at any time. In practice, security of energy supply is crucial; thus, reliability is mandatory but often neglected in optimization and only implemented subsequently employing expensive rules of thumb.

In this work, we propose an exact optimization approach to identify $(n-1)$-reliable designs for energy systems. The approach guarantees energy supply during the failure of 1 component at any time and is independent of probabilities and the selection of scenarios. $(n-1)$-reliability is also necessary to allow for maintenance of components. For problems with high computational effort, we propose the inexact but computationally efficient $(n-1^{max})$-reliability approach which also guarantees energy supply but allows overproduction. A real-world case study shows that both approaches identify reliable designs at only a small increase of the total annualized costs compared to the unreliable base case.

1. Introduction

For the design of distributed energy supply systems (DESS), mathematical optimization is a highly suitable tool (Frangopoulos et al., 2002). However, the design depends on many input parameters which are inherently uncertain, such as future energy demands or prices. In the last years, uncertainty of input data has been considered by applying robust optimization to unit commitment problems (Bertsimas et al., 2013), in the optimal design of energy systems (Akbari et al., 2014; Dong et al., 2013; Majewski et al., 2017a; 2017b; Moret et al., 2014), and in process system engineering (for a review see Grossmann et al., 2016). Recently, in process engineering, Gong and You (2017) have proposed a multi-objective two-stage adaptive robust model allowing to optimize resilience and economic objectives simultaneously. To reduce conservatism and to improve the performance of (minmax) robust optimization (Soyster, 1973), Guzman et al. (2016, 2017) propose a priori and a posteriori bounds for uncertain parameters with unknown and attributed known distributions. Ning and You (2017) extract probability distributions from uncertainty data to avoid over-conservatism. Amusat et al. (2017) quantify stochastically the effect of uncertain weather data on the design of renewable energy systems.

However, not only input parameters are uncertain but also the availability of components is uncertain (Aguilar et al., 2008). If a component fails and is not available, the energy demand has to be covered by the remaining components of the system. If the remaining energy system is not able to supply the demanded amount of energy, it is not reliable. In literature, *reliability* is defined as the probability that a system is able to provide a required function; *availability* describes the probability that an item delivers its required function during a certain time period (Aguilar et al., 2008). Since a lack of energy supply can only be avoided with certainty if the reliability of the system is 100 %, we regard a system with 100 % reliability as a reliable system in this paper.

In practice, reliability is often aimed for by heuristic rules of thumb where additional units are added (Aguilar et al., 2008). Heuristics usually result in a suboptimal design since the additional components are not part of the optimization (Andiappan et al., 2015). Therefore, developing mathematical concepts for reliability is an important research area.

For this purpose, several stochastic optimization approaches have been proposed to identify reliable designs. Sun and Liu (2015) propose a multi-period stochastic programming approach to design steam power systems. They take into account both uncertain availability of components and uncertain demands

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to aim for reliable and robust energy systems. Frangopoulos and Dimopoulos (2004) show the effect of failing components on the design of the system and the operation of the components. They employ the state-space method (for a brief introduction see Frangopoulos and Dimopoulos, 2004) to simulate partial failure of the system. Also based on the state-space method, Miryousefi Aval et al. (2015) use a two-state Markov model to design reliable building cooling, heating, and power systems and analyze the system's impact on the existing electrical power system. Abdollahzadeh and Atashgar (2017) propose a bi-objective two-stage stochastic programming model to obtain an optimal design, a maintenance strategy, and inspection intervals for supplier systems (e. g., for a wind farm). For this purpose, they minimize the system life-cycle costs and maximize the system availability. Costs and reliability of the system are also optimized by Jahromi and Feizabadi (2017) to solve the redundancy allocation problem (Barlow et al., 1965) with the aim of installing parallel redundant components. They use a gamma distribution to describe component reliability. A similar trade-off between costs and availability is considered by Ye et al. (2017). Their proposed mixed integer nonlinear programming model selects the optimal design of a reliable serial system comprising parallel components. The availability of the considered chemical process units is also described via probabilities. Andiappan et al. (2015) propose an approach based on k-out-of-n system modeling (Birnbaum, 1968) to find reliable biomass-based tri-generation systems. They determine redundancy allocation of process units with specified minimum reliability level using chance-constraint programming. Andiappan and Ng (2016) extend this k-out-of-n approach employing a generic formulation to incorporate operation strategies for the design of reliable tri-generation systems. However, employing stochastic approaches might lead to a lack of energy supply since probabilities are regarded in the optimization. Thus, stochastic approaches do not lead to a reliable system with 100 % reliability.

To avoid stochastic approaches, scenario-based analysis has been considered: Andiappan et al. (2017) have recently proposed a mixed-integer linear program to analyze the effects of unavailability of an energy conversion unit on the system flexibility without employing stochastic optimization. Instead, they use input-output modeling (Leontief, 1936) based on disruption scenarios to identify insufficient flexibilities and deduce a step-by-step guide to retrofit an existing energy system. Another exact algorithm, also based on scenarios, has been proposed by Caserta and Voß (2015). They incorporate redundancies by transforming the reliability redundancy allocation problem into a multiple-choice knapsack problem and employ a multi-period approach to take potential scenarios into account. Aguilar et al. (2008) consider maintenance and failure scenarios to design reliable energy supply systems regarding the largest one, two, or more components to be turned off. Thus, for the considered scenarios, the resulting energy supply system is able to provide the necessary amount of energy even if a component fails during maintenance of other components. Considering scenarios of failure implies that a failure of any component not included in the scenarios might lead to a lack of energy supply. Thus, trustworthiness of reliability calculations always depends on the correct selection of scenarios.

In power systems engineering, reliable electricity supply is crucial and an active research area. Thus, many approaches have been proposed to increase reliability employing optimization: Ruiz and Conejo (2015) and Mínguez and García-Bertrand (2016) employ adaptive robust optimization to solve the transmission expansion planning problem. A scenario-based approach is proposed by Alguacil et al. (2009) evaluating the trade-off between investment cost reduction and vulnerability of the transmission network against attacks using a weighted objective function. Ruiz et al. (2009) enforce reserve requirements in a stochastic formulation to compensate the limited representation of uncertainty by the selection of scenarios. In their paper, uncertainty refers to both load uncertainty and generation unreliability. Choi et al. (2005) propose a methodology based on probabilistic reliability criteria to minimize cost of transmission system expansion. In a later paper, Choi et al. (2006) additionally regard costs of outages. Another well-known approach in power systems engineering is $(n-K)$-reliability. $(n-K)$-reliability ensures reliability of a power system with n components during the failure of maximal K components. For K being 1, stochastic approaches have been developed to find $(n-1)$-reliable solutions for network expansion and transmission switching in reasonable time, e. g., by Wiest et al. (2018) and Hedman et al. (2010). For the German power grid, $(n-1)$-reliability represents an adequate reliability of supply (Berndt et al., 2007). Employing robust optimization, $(n-K)$-reliability is employed in contingency-constraint transmission expansion planning by Moreira et al. (2015) and additionally regarding load uncertainty by Hong et al. (2017). In contingency-constraint unit commitment, Wang et al. (2013) and Street et al. (2011) introduce $(n-K)$-reliability using a robust formulation. Street et al. (2011) propose an exact approach which does not depend on the cardinality of the uncertainty set. However, this approach is based on the assumption of single-bus unit commitment problem in which only upward reserves need to be considered. This shortcoming has been overcome by Street et al. (2014) who also take into account failure in the transmission network. To solve the extended model, a Benders' decomposition approach (Benders, 1962) is necessary. The concept of reliability is also crucial for DESS. In DESS, several components are usually operated to supply the desired energy demand. Failure of any component could always occur and the remaining components would need to compensate the loss to ensure reliable operation.

In this paper, we present a rigorous approach *ensuring* reliability in the design of DESS during the failure of 1 component. Therefore, we newly introduce $(n-1)$-reliability into the optimal design of DESS. The proposed $(n-1)$-reliability approach follows the idea of approaches for reliable power systems. However, for DESS optimization, additional types of final energy need to be considered, e. g., thermal energy. Furthermore, we take into account the possible upwards and downwards reserves of supplied energy by the components remaining available during failure. Thus, the proposed rigorous approach does not exclude possibly optimal solutions a priori. Hence, our $(n-1)$-reliable approach extends existing approaches from power system engineering. Moreover, the obtained $(n-1)$-reliable design is independent of selecting failure scenarios and independent of failure probabilities. Instead, we consider a possible failure of 1 arbitrary component at any time. The identified reliable design is able to cover all energy demands exactly during maintenance or failure. For problems with high computational effort, we propose an inexact but computationally efficient approach called $(n-1^{\max})$-reliability. In this alternative approach, we ensure sufficient energy supply during the failure of any component while allowing overproduction. Allowing overproduction reduces the optimization problem to the analysis of failure of the largest component and is therefore called $(n-1^{\max})$-reliability.

The remaining article is structured as follows: First, we introduce the nominal problem without considering any reliability in Section 2.1. In the following Sections 2.2 and 2.3, we introduce our exact $(n-1)$-reliable approach and our inexact but computationally efficient $(n-1^{\max})$-reliable approach. The approaches are applied to a real-world case study in Section 3. We summarize and conclude our research in Section 4.

2. Reliable design of distributed energy supply systems

Reliability of distributed energy supply systems (DESS) is mandatory in practical applications. Security of energy supply and allowing for maintenance of components at any time is crucial for DESS since a lack of energy supply can lead to incalculable costs, e. g., due to shutdown of a plant or accidents. To guarantee sufficient energy supply, we propose two reliability approaches. We present the approaches for a tri-generation system providing electricity, heating, and cooling energy. However, the concept can be generalized to other demands. In our context, we assume that electricity demands can always be covered by the electricity grid; thus, we focus on covering heating and cooling demands during failure of components. Before presenting the reliability approaches, we state the nominal optimization problem for DESS without reliability considerations in the next section.

2.1. The nominal model of DESS design

The considered model of DESS has been published in our earlier work (Voll et al., 2013). The optimization problem is formulated as a mixed integer linear program (MILP) and identifies an optimal design (structure of DESS, sizing of components) and an optimal operation of the designed DESS. In this paper, each installed component transforms input energy with a constant efficiency to output energy. Operation of components is possible between installed thermal power and a technology-specific minimal part load. All newly installed components have capacity-dependent investment costs which are piece-wise linearized. For each technology, the automated superstructure-generation approach from Voll et al. (2013) is applied which successively increases the number of components to build up a superstructure which contains the optimal design and is not oversized. The largest possible superstructure resulting from this successive approach contains a predefined maximum number of boilers, combined heat and power engines, compression chillers, and absorption chillers. In the case study, the maximal number of units per technology is set to 10. In the model proposed by Voll et al. (2013), no failure of components is taken into account; and thus, no reliability can be ensured. In the following, we call this model *nominal* to clearly distinguish between the model without considering any failure and the application of reliability approaches.

As objective function, we choose the total annualized costs *TAC* to optimize the DESS:

$$TAC = \sum_{t \in T} \left[\Delta\tau_t \left(p^{gas} \cdot \dot{U}_t^{gas,buy} + p^{el,buy} \cdot \dot{U}_t^{el,buy} - p^{el,sell} \cdot \dot{V}_t^{el,sell} \right) \right] + \sum_{k \in \mathcal{K}} \left(\frac{1}{PVF} + p_k^m \right) \cdot CAPEX_k.$$

Here, p^{gas}, $p^{el,\ buy}$, and $p^{el,\ sell}$ are prices for purchasing gas and for buying and selling electricity, respectively. $\dot{U}_t^{gas,buy}$ and $\dot{U}_t^{el,buy}$ represent the input energy flows of gas and electricity, and $\dot{V}_t^{el,sell}$ the output energy flow of electricity in time step $t \in T$. The set T contains all time steps t and their corresponding lengths are denoted by $\Delta\tau_t$. Annual maintenance costs are given as share p_k^m of the investment costs $CAPEX_k$ of each component k within the set of all components $\mathcal{K}$ which might be installed. The investment costs $CAPEX_k$ are annualized using the present value factor (Broverman, 2010)

$$PVF = \frac{(i+1)^h - 1}{(i+1)^h \cdot i}$$

with an interest rate i and a time horizon h. Instead of the total annualized costs *TAC*, any other objective function can be chosen (for a discussion see Hennen et al., 2017).

Besides the influence of the objective function on the optimal solution, the optimization depends on physical constraints—precisely, energy balances. For the nominal problem, the energy balances are given by:

$$\sum_{k \in \mathcal{K}} \dot{V}_{kt} = \dot{E}_t \qquad \forall t \in T\,. \tag{1}$$

$\dot{V}_{kt}$ represents the supplied energy flow of component k in time step t. The energy demands in time step t are given by $\dot{E}_t$. $\dot{E}_t$ represents both heating demands $\dot{E}_t^h$ and cooling demands $\dot{E}_t^c$. For heating, the energy demand comprises not only the heating requirements on the industrial site $\dot{E}_t^{h,s}$, but also the necessary energy $\sum_{k \in AC} \frac{\dot{V}_{kt}}{\eta_k}$ for running the absorption chillers $k \in AC$ with coefficient of performance (COP) η_k:

$$\dot{E}_t^h = \dot{E}_t^{h,s} + \sum_{k \in AC} \frac{\dot{V}_{kt}}{\eta_k} \qquad \forall t \in T\,. \tag{2}$$

For more details on the model see Voll et al. (2013) or Majewski et al. (2017b).

2.2. $(n-1)$-reliability: an exact approach for the optimal reliable design

The idea of $(n-1)$-reliability is that the designed system can compensate the failure of any component out of n components at any time—but only 1 at the same time. Thus, a failure of 1 component during maintenance of another might still lead to a lack of energy supply. In other words, we aim for an $(n-1)$-out-of-n system (Birnbaum, 1968) in which $n-1$ components out of n need to be fully available to guarantee a 100 % reliable system. To model $(n-1)$-reliability, we consider all possible scenarios for a failure of 1 single component. To implement $(n-1)$-reliability, we need additional energy balances for heating and cooling circuits describing the assumption that any component might fail at any time and cannot provide energy:

$$\sum_{k \in \mathcal{K}^h \setminus \{k'\}} \dot{V}_{kt}^{k'} = \dot{E}_t^{h,s} + \sum_{k \in AC \setminus \{k'\}} \frac{\dot{V}_{kt}^{k'}}{\eta_k} \qquad \forall t \in T \quad \forall k' \in \mathcal{K} \tag{3}$$

$$\sum_{k \in \mathcal{K}^c \setminus \{k'\}} \dot{V}_{kt}^{k'} = \dot{E}_t^c \forall t \in T \qquad \forall k' \in \mathcal{K}\,. \tag{4}$$

The set of components $\mathcal{K}$ is divided into heating $\mathcal{K}^h$ and cooling components $\mathcal{K}^c$. Every possibly failing component $k' \in \mathcal{K}$ leads to a new set of energy balances. To formulate these additional energy balances, we introduce new operation variables $\dot{V}_{kt}^{k'}$ which specify the adapted output of component k in time step t if component k' fails. The new operation variables are determined such that the demands can still be covered exactly with the remaining components $\mathcal{K} \setminus \{k'\}$.

Since the additional variables $\dot{V}_{kt}^{k'}$ only appear in the constraints and not in the objective function, they only impact the design of the DESS, i. e., the selected components and their sizing. The objective function remains without any changes which allows replacing the criterion for the optimization easily.

The additional energy balances lead to many additional equality constraints and the problem size scales up polynomially in two variables: The number of additional constraints increases proportionally with the number of potentially installed components $|\mathcal{K}|$ times the number of considered time steps $|T|$.

The problem of $(n-1)$-reliability can also be formulated using robust optimization. For this purpose, adjustable robustness (Ben-Tal et al., 2004) can be employed to describe the adaption of DESS to a failing component: The remaining components adjust their amount of provided energy such that the demands can still be fulfilled:

$$\sum_{k\in\mathcal{K}^h} \dot{V}_{kt}(\xi_{kt}) - \xi_{kt}\dot{V}_{kt}(\xi_{kt}) = \dot{E}_t^{h,s} + \sum_{k\in AC} \frac{\left(\dot{V}_{kt}(\xi_{kt}) - \xi_{kt}\dot{V}_{kt}(\xi_{kt})\right)}{\eta_k} \quad (5)$$

$$\forall t \in T \quad \forall \xi_{kt} \in \mathcal{U}$$

$$\sum_{k\in\mathcal{K}^c} \dot{V}_{kt}(\xi_{kt}) - \xi_{kt}\dot{V}_{kt}(\xi_{kt}) = \dot{E}_t^c \quad (6)$$

$$\forall t \in T \quad \forall \xi_{kt} \in \mathcal{U}\,.$$

The additional operation variables $\dot{V}_{kt}(\xi_{kt})$ depend on the scenario ξ_{kt} with $\xi_{kt} = 1$ describing the failure of component k in time step t. The corresponding uncertainty set is given by

$$\mathcal{U} := \left\{\xi'_{kt} : \left(\xi'_{kt}\right)_{k\in\mathcal{K},t\in T} \in \{0,1\}^{|\mathcal{K}|\times|T|}, \sum_{k\in\mathcal{K}} \xi'_{kt} = 1, \forall t \in T\right\}. \quad (7)$$

The sum $\sum_{k\in\mathcal{K}} \xi'_{kt} = 1$ ensures that only 1 component fails at the same time.

$(n-1)$-reliability can be extended such that the failure of more than 1 component is taken into account as done for electricity grids by Street et al. (2014). For DESS, using the notation of Eqs. (3) and (4), the following equations need to be added to the nominal problem:

$$\sum_{k\in\mathcal{K}^h\setminus\{\mathcal{K}'\}} \dot{V}_{kt}^{\mathcal{K}'} = \dot{E}_t^{h,s} + \sum_{k\in AC\setminus\{\mathcal{K}'\}} \frac{\dot{V}_{kt}^{\mathcal{K}'}}{\eta_k} \qquad \forall t \in T \quad \forall \mathcal{K}' \in \mathcal{P}(\mathcal{K}) \quad (8)$$

$$\sum_{k\in\mathcal{K}^c\setminus\{\mathcal{K}'\}} \dot{V}_{kt}^{\mathcal{K}'} = \dot{E}_t^c \qquad \forall t \in T \quad \forall \mathcal{K}' \in \mathcal{P}(\mathcal{K})\,. \quad (9)$$

where $\mathcal{K}'$ is a subset of the power set $\mathcal{P}(\mathcal{K})$ including all combinations of possibly failing heating and cooling components $\mathcal{K}$. The cardinality of failing components at the same time can be limited, i. e., $|\mathcal{K}'| \leq K$. This formulation guarantees $(n-K)$-reliability. $(n-K)$-reliability leads to an exponential increase of the number of additional equations proportional to $\sum_{i=1}^{K}\left(\binom{|\mathcal{K}|}{i}\cdot|T|\right)$ and thus of computational time. For DESS, $(n-1)$-reliability, i. e., $K = 1$, is the most basic and commonly applied method of $(n-K)$-reliability in engineering practice. Eqs. (8) and (9) are equivalent to Eqs. (3) and (4) when the failure of only 1 component is considered. Considering the failure of 1 arbitrary component is often sufficient (Aguilar et al., 2008). Thus, we focus in this paper on $(n-1)$-reliable design of energy systems.

However, employing $(n-1)$-reliability already leads to an increased problem size which might result in high computational times. Thus, we propose an alternative, computationally fast approach while still ensuring sufficient energy supply; however, the solutions may lead to overproduction during failure of a component.

2.3. $(n-1^{\max})$-reliability: an inexact approach for the optimal design ensuring sufficient energy supply

The idea of $(n-1^{\max})$-reliability is to be able to supply at least any required heating and cooling demand during a failure of any single component at any time but to allow for overproduction. Overproduction describes an operational state in which more energy is supplied than demanded. If overproduction is allowed, it is sufficient to consider only the failure of the largest cooling or heating component to cover the failure of any component: A larger component can always replace a smaller component. The only limitation could arise from minimal part-load constraints for the larger component. Here, this limitation is overcome by allowing overproduction during failure. Any excess energy produced during overproduction has to be wasted, i. e., released to the environment. Alternatively, a component could be operated below the specified minimal part-load operation. To be able to supply at least all heating demands $\dot{E}_t^h$ and cooling demands $\dot{E}_t^c$ during failure of *1 arbitrary* component, we ensure sufficient capacity during the failure of the *largest* component:

$$\sum_{k\in\mathcal{K}^{h/c}} \dot{V}_k^N - \dot{V}_{\mathcal{K}^{h/c}}^{\max} \geq \dot{E}_t^{h/c} \qquad \forall t \in T\,. \quad (10)$$

Again, $\mathcal{K}^h$ denotes heating components and $\mathcal{K}^c$ cooling components. $\dot{V}_k^N$ represents the installed thermal power of component k and $\dot{V}_{\mathcal{K}^{h/c}}^{\max}$ the maximal deficit in heating/cooling supply induced due to failure of 1 component. The maximal deficit $\dot{V}_{\mathcal{K}^{h/c}}^{\max}$ is defined as the maximal installed heating/cooling energy of 1 single component:

$$\dot{V}_{\mathcal{K}^{h/c}}^{\max} := \max\left\{\dot{V}_k^N \,\middle|\, k \in \mathcal{K}^{h/c}\right\} \quad (11)$$

which can be reformulated by

$$\dot{V}_{\mathcal{K}^{h/c}}^{\max} \geq \dot{V}_k^N \qquad \forall k \in \mathcal{K}^{h/c}\,. \quad (12)$$

Since the heating demands $\dot{E}_t^h$ also depend on the operation of the absorption chillers (see Eq. (2)), a failure in a cooling circuit might also affect the heating circuit: A failing compression chiller could be replaced by absorption chillers. The replacing absorption chillers increase the heating demand. This dependency needs to be taken into account for reliability:

$$\sum_{k\in\mathcal{K}^h} \dot{V}_k^N \geq \dot{E}_t^{h,s} + \sum_{k\in AC} \frac{\dot{V}_{kt}}{\eta_k} + \dot{E}_{AC}^c \qquad \forall t \in T\,. \quad (13)$$

The installed heating capacity $\sum_{k\in\mathcal{K}^h} \dot{V}_k^N$ needs to cover at least the current total heating demand of each time step t, i. e., the sum of $\dot{E}_t^{h,s}$ and $\sum_{k\in AC} \frac{\dot{V}_{kt}}{\eta_k}$. $\dot{E}_{AC}^c$ adds the maximal heating supply due to failure of a cooling component:

$$\dot{E}_{AC}^c = \min\left\{\max_{k\in CC} \frac{\dot{V}_k^N}{\eta_{AC}^{\min}}\;;\; \sum_{k\in AC} \frac{\dot{V}_k^N}{\eta_k}\right\}. \quad (14)$$

Here, the first term, $\frac{\dot{V}_k^N}{\eta_{AC}^{\min}}$, describes the heating energy needed to replace the largest compression chiller by absorption chillers considering their worst coefficient of performance $\eta_{AC}^{\min}$. The second term, $\sum_{k\in AC} \frac{\dot{V}_k^N}{\eta_k}$ corresponds to the maximal heating energy required by the absorption chillers. We propose an MILP reformulation of Eq. (14) in Appendix A.1. An even tighter but more complex approximation of the maximal additional heating demand $\dot{E}_{AC}^c$ is given in Appendix A.2.

A failing heating component $k \in \mathcal{K}^h$ might also affect a cooling circuit since absorption chillers depend on the provided heating energy. However, this coupling of circuits is already regarded by employing Eq. (2).

To model the ability to cover at least the required demands during the failure of 1 arbitrary component at any time, we only need to include Eqs. (10) and (14) to the optimization problem. Since only constraints are added to the nominal problem, applying the $(n-1^{\max})$-reliable approach allows changing the objective function as easily as in the $(n-1)$-reliable approach.

The advantage of $(n-1^{\max})$-reliability compared to $(n-1)$-reliability is that the number of additional constraints is significantly reduced. Here, the number of additional constraints does not depend on the product of number of time steps $|T|$ and components $|\mathcal{K}|$ but only on the sum. Eq. (10) involves $2\cdot|T|$ additional equations which can be further reduced, since the maximal cooling demand is already known and thus not time dependent. Since the constraints for calculating the maximal deficit (Eq. (12)) only depend on the set of potentially installed components $\mathcal{K} = \mathcal{K}^h \cup \mathcal{K}^c$ but not on the set of time steps T, the number of additional constraints is proportional to the number of potentially installed components $|\mathcal{K}|$. The number of additional equations for introducing Eq. (14) is proportional to the number of cooling components $|\mathcal{K}^c|$ (see Appendix A.1). Considering all additional equations, the problem complexity increases proportionally to the sum of number of time steps and components $|T| + |\mathcal{K}|$. As a result, the expected increase in computational time compared to the nominal problem is low—at the cost of potential overproduction.

3. Case study

In this section, the proposed reliability approaches are applied to a real-world case study of an industrial park. The case study is based on a nominal design problem of a distributed energy supply system (DESS) published by Voll et al. (2013). The problems are implemented in GAMS 24.7.3 (McCarl and Rosenthal, 2016) and CPLEX 12.6.3.0 (IBM Corporation, 2015) is used to solve the problems to machine accuracy on a computer with 3.24 GHz and 64 GB RAM employing 4 threads.

3.1. Description of the real-world industrial site

The analyzed industrial park comprises a heating system and a cooling system which is divided into two separated cooling circuits: *Site A* and *Site B*. The time-varying heating, cooling, and electricity demands are time-aggregated. Additionally, peak demands are considered (see Fig. 1 and Table B.1).

The energy demands can be covered by installing combined heat and power engines *CHP*, boilers *B*, absorption chillers *AC*, and compression chillers *CC*. Minimal part-load operation of components is taken into account based on equipment data sheets (Voll et al., 2013): The minimal part load of combined heat and power engines is 50 % of the installed thermal power. All remaining technologies can operate between 20 % of the installed power and full load.

We optimize the total annualized costs *TAC* as objective function using an interest rate i of 8 % and a time horizon h of 4 years to calculate the present value factor *PVF*. Furthermore, we assume a "green field" without existing energy system components on the industrial site. However, the proposed reliability approaches can also be employed for reliable optimal retrofit of DESS. Expected solutions would comprise existing components with possibly poor efficiencies as spare components to prevent additional investment costs.

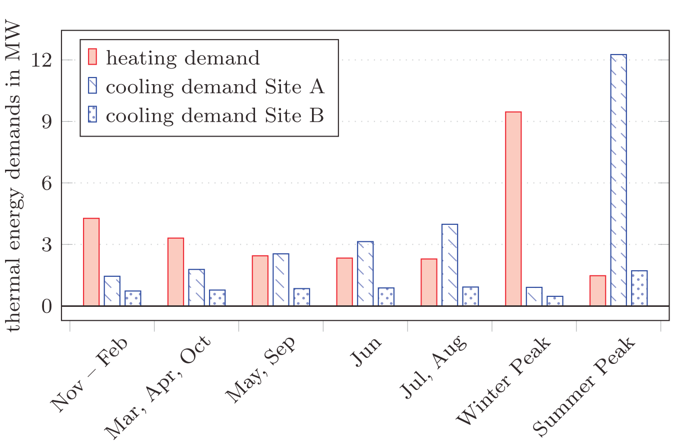

Fig. 1. Heating and cooling demands of the real-world industrial site.

3.2. Reliable designs for the distributed energy supply system

To evaluate the reliability approaches introduced in Sections 2.2 and 2.3, we consider a pragmatic approach from engineering practice as benchmark: After identifying an optimal design, the largest component of each heating and cooling circuit is installed twice to increase reliability of the DESS when allowing overproduction. We call this approach *(2 × max)-reliability*. The corresponding design is called *(2 × max)-reliable design* $d^{(2\times\max)}$.

In the following, we compare results of the nominal problem to the heuristic (2 × max)-reliable approach, the inexact $(n-1^{\max})$-reliable approach (see Section 2.3), and the exact $(n-1)$-reliable approach (see Section 2.2). Fig. 2 shows the selected components and their sizing (i. e., the optimal design) for the nominal problem and the three reliability approaches.

The (2 × max)-reliable design $d^{(2\times\max)}$ has the same components as the nominal design d^{nom} but includes twice the largest component of each circuit. This approach usually leads to high investment costs and, in general, failure of a small component might lead to high overproduction. In the presented case study, the (2 × max)-reliable design $d^{(2\times\max)}$ can compensate the failure of any component and is able to fulfill the $(n-1)$-reliable energy balances (Eqs. (3) and (4)). In general, there could be cases where the (2 × max)-reliable design is not fully reliable: If an absorption chiller is installed twice and the only compression chiller fails, the additionally needed heating supply might not be covered by the installed components. However, this scenario seems rather fabricated and unlikely in practice.

Employing $(n-1^{\max})$-reliability and $(n-1)$-reliability leads to nearly identical reliable designs: For most component types, the designs comprise more and smaller components than the nominal and the (2 × max)-reliable design. As a result, the designs have a higher system flexibility. In general, the $(n-1^{\max})$-reliable design $d^{(n-1^{\max})}$ cannot guarantee to cover energy demands exactly and might lead to overproduction if small components fail. Such designs are discussed in Section 3.3. Here, no overproduction occurs

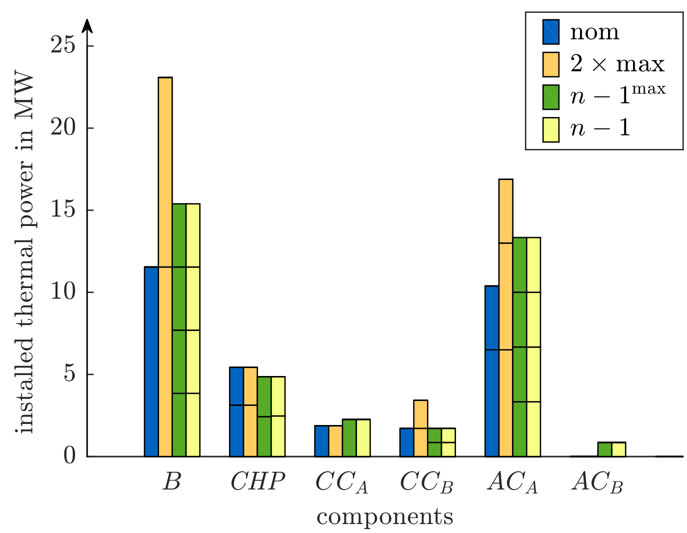

Fig. 2. Installed components and their installed thermal capacities for each approach: components of nominal design d^{nom} (without considering failure), of (2 × max)-reliable design $d^{(2\times\max)}$ (doubling the largest component of the nominal design for each circuit), of $(n-1^{\max})$-reliable design $d^{(n-1^{\max})}$ (inexact approach allowing failure of any component but allowing overproduction), and of $(n-1)$-reliable design $d^{(n-1)}$ (exact method allowing failure of any component) for each technology from left to right; *B* boiler, *CHP* combined heat and power engine, CC_A and CC_B compression chillers, and AC_A and AC_B absorption chillers installed on Site A and Site B, respectively.

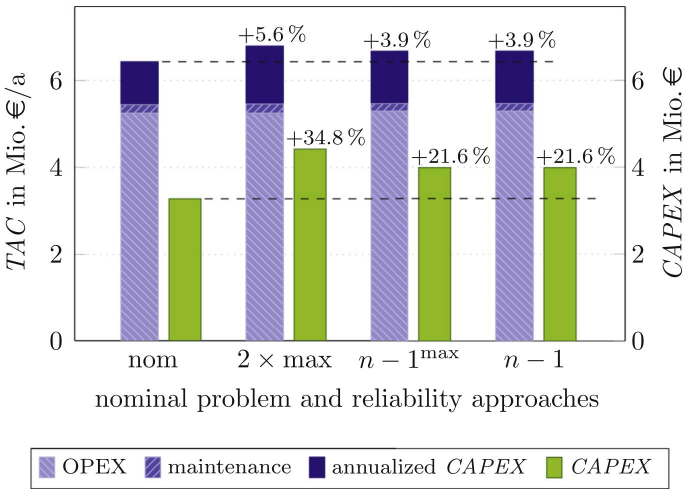

Fig. 3. Total annualized costs *TAC*, operational costs *OPEX*, and (annualized) investment costs *CAPEX* of the nominal problem and the reliability approaches.

during failure, if the $(n-1^{\max})$-reliable design $d^{(n-1^{\max})}$ is implemented.

The increase in costs compared to the nominal problem differs for the three reliability approaches (Fig. 3).

The total annualized costs $TAC^{(2\times\max)}$ for the $(2\times\max)$-reliable design $d^{(2\times\max)}$ lie 5.6 % above the costs for the nominal design d^{nom}. Even though the $(2\times\max)$-reliable design $d^{(2\times\max)}$ involves the highest increase in costs, reliability for all possible failures cannot be guaranteed in general without possibly high overproduction. The total annualized costs for the designs of the $(n-1^{\max})$-reliable approach and of the $(n-1)$-reliable approach are only 3.9 % higher than costs for the nominal design d^{nom}. Thus, both $(n-1^{\max})$-reliability and $(n-1)$-reliability save 30.4 % of the additional costs for a reliable system.

Operational costs *OPEX* are nearly identical for all designs with only 0.8 % variation. The values take 5.25 Mio. €/a for the nominal problem and $(2\times\max)$-reliability, and 5.29 Mio. €/a for the proposed reliability approaches. Significant differences can be observed in the investment costs *CAPEX* (Fig. 3): The nominal design induces 3.28 Mio. € investment costs, whereas the $(2\times\max)$-reliable design induces 4.42 Mio. € corresponding to an increase of 34.8 %. In contrast, the $(n-1^{\max})$-reliable and $(n-1)$-reliable designs involve only lower additional investment costs compared to the nominal design d^{nom}, i. e., 0.71 Mio. € corresponding to an increase of 21.6 %. Compared to the heuristic $(2\times\max)$-reliable approach, the $(n-1^{\max})$-reliable and $(n-1)$-reliable designs save 37.7 % of the increase in investment costs. Especially in industry, low up-front costs are attractive.

The results show that cost-efficient reliable design options can be identified with both proposed reliability approaches.

3.3. Assessment of reliability approaches

In order to assess our approaches, we vary the demand time series by ±5 % using latin-hypercube sampling (McKay et al., 2000). Even for such slight perturbations, the obtained optimal solutions and the performance of MILP solvers can vary dramatically. Thus, we consider 10 varied instances of our original demand time series in the following.

For the 10 instances and for the original time series, Table 1 shows the deviation of the total annualized costs from the nominal total annualized costs TAC^{nom} as well as the CPU time for computation.

The results show that the costs for the $(n-1^{\max})$-reliable design $d^{(n-1^{\max})}$ and the $(n-1)$-reliable design $d^{(n-1)}$ are equal for all instances. Both proposed reliability approaches reduce the total annualized costs compared to the heuristic $(2\times\max)$-reliable approach. For all instances, reliability can be achieved with a low increase of total annualized costs, i. e., 3.9 % on average. Furthermore, the shares of operational costs, maintenance costs, and investment costs are similar to the original time series (Fig. 3). The results show that the increase in investment costs for a reliable design can be reduced by up to 45.7 % compared to the heuristic $(2\times\max)$-reliability. The operational costs and the investment costs including their corresponding variation for the instances are presented in Fig. C.1. The variation of the demands shows that the expected additional costs for a reliable design are low if $(n-1^{\max})$-reliability or $(n-1)$-reliability is employed.

Computational times of the different approaches, also listed in Table 1, show the advantage of the $(n-1^{\max})$-reliable approach: The $(n-1^{\max})$-reliable approach is more than 19.6 times faster than the $(n-1)$-reliable approach. Moreover, the $(n-1^{\max})$-reliable approach is only maximal 2.7 times slower than the $(2\times\max)$-reliable approach which identifies designs involving high additional costs. Thus, the $(n-1^{\max})$-reliable approach clearly outperforms $(2\times\max)$-reliability and $(n-1)$-reliability in both computational time and expected costs for a reliable design. However, the $(n-1^{\max})$-reliable approach might identify solutions leading to overproduction during failure.

The differing computational times of the approaches are the result of their respective model scales. For the original instance, the numbers of variables and equations after presolve are given in Table 2. The numbers for $(2\times\max)$-reliability comprise values for the nominal problem plus numbers for additional optimization

Table 1
Deviation of the total annualized costs from the nominal total annualized costs TAC^{nom} as well as the CPU time for the original demand time series and the additional 10 instances.

Instance	Increase of TAC^{nom} in %			CPU time in s			
	$2\times\max$	$n-1^{\max}$	$n-1$	nom	$2\times\max$	$n-1^{\max}$	$n-1$
original	5.6	3.9	3.9	5.7	5.8	15.3	299.3
1	5.7	3.9	3.9	6.0	6.0	15.3	506.8
2	5.3	3.8	3.8	5.6	5.7	15.2	388.1
3	5.6	3.9	3.9	5.6	5.6	15.0	555.7
4	5.6	3.9	3.9	5.6	5.6	15.1	556.4
5	5.9	4.0	4.0	5.7	5.8	15.2	369.4
6	5.7	3.9	3.9	5.7	5.8	14.9	315.7
7	5.6	3.9	3.9	5.6	5.7	15.0	492.1
8	5.5	3.7	3.7	5.7	5.7	15.1	372.6
9	5.7	3.8	3.8	5.6	5.7	15.2	407.8
10	5.6	3.9	3.9	5.7	5.8	15.1	559.3

Table 2
Number of equations, continuous variables, and binary variables of the nominal problem and the three reliability approaches.

Approach	Equations	Continuous variables	Binary variables
nom	657	264	102
$2 \times \max$	678	278	111
$n-1^{\max}$	1435	491	227
$n-1$	31852	9220	4774

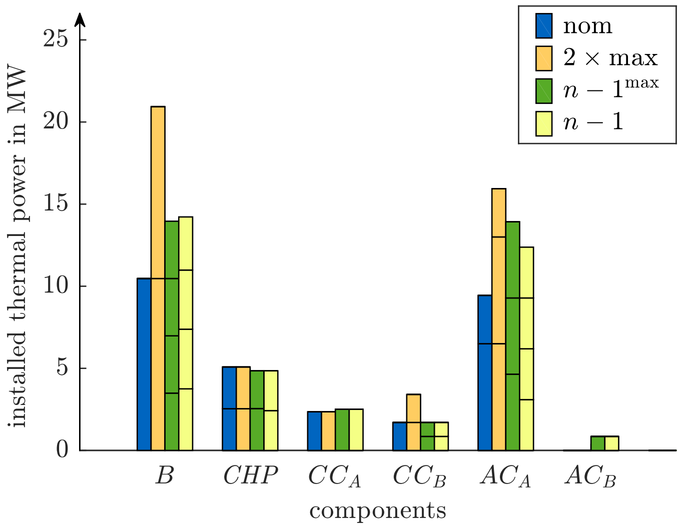

Fig. 4. Selected components and their installed thermal power for instance 7 of the varied demand time series; components of nominal design d^{nom}, of $(2 \times \max)$-reliable design $d^{(2\times\max)}$, of $(n-1^{\max})$-reliable design $d^{(n-1^{\max})}$, and of $(n-1)$-reliable design $d^{(n-1)}$ for each technology from left to right; *B* boiler, *CHP* combined heat and power engine, CC_A and CC_B compression chillers, and AC_A and AC_B absorption chillers installed on Site A and Site B, respectively.

of operation with the added largest unit. The $(n-1^{\max})$-reliability approach additionally involves 2 indicator variables in GAMS for selecting whether Eqs. (A.1) or (A.4) is active.

Analyzing the instances of the demand time series shows that 4 of the 10 instances lead to $(n-1^{\max})$-reliable designs $d^{(n-1^{\max})}$ which cannot cover the energy demands exactly if 1 component fails. We call these solutions *inexact*. All inexact $(n-1^{\max})$-reliable designs suffer from the same shortcoming: If a small component fails, overproduction is necessary to guarantee sufficient energy supply in the time step with minimal cooling demand. Table 3 lists all possibly occurring overproduction induced by inexact solutions.

In order to analyze the reason for overproduction, we take a closer look at the design of a typical inexact solution of $(n-1^{\max})$-reliability. Fig. 4 shows the designs for instance 7 of the varied demand time series. The corresponding demands of instance 7 used as input for the optimization are listed in Table B.2.

Here, we focus on the differences between the $(n-1^{\max})$-reliable design $d^{(n-1^{\max})}$ and the $(n-1)$-reliable design $d^{(n-1)}$. In the $(n-1^{\max})$-reliable design $d^{(n-1^{\max})}$, the boilers are slightly smaller than in the $(n-1)$-reliable design $d^{(n-1)}$. However, the actual difference leading to overproduction is the larger sizing and the smaller number of absorption chillers on Site A (AC_A) in the $(n-1^{\max})$-reliable design $d^{(n-1^{\max})}$: If the small and unique compression chiller on Site A (CC_A) fails, the minimal part load of the replacing absorption chillers is too high to cover the minimal cooling demand exactly. However, in instance 7, only 49.1 kW of cooling are additionally produced which corresponds to 5.6 % of the needed cooling supply in this time step (Table 3). The occurring overproduction can be avoided by running 1 absorption chiller in part load with 18.9 % of the nominal installed power.

Table 3
Necessary overproduction for all instances leading to inexact $(n-1^{\max})$-solutions to cover energy demands; overproduction only occurs in the time step with minimal cooling demand.

Instance	Overproduction in kW	Overproduction in %
4	24	2.6
5	58.6	6.4
7	49.1	5.6
9	107.3	12.3

For the remaining inexact solutions, the reason for overproduction is exactly the same. To avoid overproduction for the remaining inexact solutions, it is sufficient to decrease the part load of the absorption chiller to a minimum of 17.8 % of the nominal installed power. This required part load is below the specified minimal part load of 20 % for the selected absorption chillers (see Section 3.1). However, in practice, a slight decrease of the minimal part load or a slight overproduction for a short time might be justifiable in emergencies.

The analysis shows that overproduction is only a minor problem in the considered case study. Nevertheless, we want to introduce an idea how the $(n-1^{\max})$-reliable approach can easily be further improved if desired: Just as for the failure of the largest component ensuring covering maximal demands, it is possible to add further constraints ensuring that the smallest demands can be covered exactly without the component with the smallest part load. As in the original $(n-1^{\max})$-reliable approach, both heating and cooling circuits need to be considered in the constraints. Such an extended approach would reduce the risk of solutions leading to overproduction; however, inexact solutions involving overproduction might still occur. The case study shows that the $(n-1^{\max})$-reliable approach performs well even without the introduced extension.

The increase in computational time compared to the nominal problem is low. Moreover, $(n-1^{\max})$-reliability enables the design of flexible DESS ensuring sufficient energy supply during failure with only slightly increased total annualized costs.

4. Conclusions

In the design of distributed energy supply systems (DESS), reliability of the system is an important but often neglected issue. We propose an approach to identify exact $(n-1)$-reliable designs for DESS. Our $(n-1)$-reliable approach ensures reliable energy supply during the failure of 1 component at any time. Thus, the approach also enables to maintain components at any time if no failure occurs simultaneously.

The number of additional constraints in the $(n-1)$-reliable approach depends on the product of the number of potentially installed components and the number of time steps leading to a polynomial increase of the problem complexity. Thus, the exact algorithm might lead to high computational effort. For this reason, we propose also a time-efficient inexact approach, called $(n-1^{\max})$-reliability which ensures that all demands can still be covered if 1 component fails but allows for overproduction. The time-efficient $(n-1^{\max})$-reliable approach involves only few additional constraints and the increase in computational time is low. Therefore, $(n-1^{\max})$-reliability would allow increasing model accuracy or coupling reliability with other optimization approaches with increased computational time, e. g., multi-objective optimization.

An industrial real-world case study shows that pragmatic heuristics from industry applications, such as installing redundant components by rules of thumb, lead to unnecessary high additional investment costs for reliability. Employing both the proposed exact and inexact reliability approaches, we obtain well-performing solutions during failure and achieve savings of 30.4 % in the additional total annualized costs compared to pragmatic heuristics from industry applications. The increase in the investment costs for a reliable design can even be reduced by 37.7 % employing the proposed reliability approaches. Reliable designs of the proposed $(n-1)$-reliable approach and the $(n-1^{\max})$-reliable approach lead to an increase in total annual costs of only 3.9 % compared to the total annualized costs of an optimal system design without providing any reliability. Thus, our proposed approaches enable to identify optimal reliable designs for distributed energy supply systems at low additional costs. The exact $(n-1)$-reliable approach can ensure reliability while $(n-1^{\max})$-reliability provides a well-performing inexact approach which is easy to implement and which involves low additional computational effort.

Employing the proposed reliability approaches ensures that any component can be maintained at any time and ensures sufficient energy supply during failure of any component. Thereby, unpredictable costs can be avoided and security in industrial applications can be improved significantly.

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Appendix A. The maximal additionally needed heating supply due to failure of a cooling component

A.1. Reformulation of equation constraining the maximal additional heating $\dot{E}^c_{AC}$

To obtain an MILP, we reformulate Eq. (14) by:

$$\dot{E}^c_{AC} \geq x \cdot \frac{\dot{V}^{\max}_{CC}}{\eta^{\min}_{AC}} \tag{A.1}$$

$$\eta^{\min}_{AC} \leq \eta_k \qquad \forall\ k \in AC \tag{A.2}$$

$$\dot{V}^{\max}_{CC} \geq \dot{V}^N_k \qquad \forall\, k \in CC \tag{A.3}$$

$$\dot{E}^c_{AC} \geq y \cdot \sum_{k \in AC} \frac{\dot{V}^N_k}{\eta_k} \tag{A.4}$$

$$x + y = 1 \tag{A.5}$$

$$x, y \in \{0, 1\}\,. \tag{A.6}$$

The bilinear products with the binary variables x and y in Eqs. (A.1) and (A.4) can be linearized, e. g., using Glover's linearization (see Glover, 1975). The binary variables x and y activate and deactivate the lower bound of the maximal additionally needed heating supply for absorption chillers during the failure of a cooling component $\dot{E}^c_{AC}$. Since only 1 bound needs to be active (see Eq. (A.5)), the maximal additional heating supply $\dot{E}^c_{AC}$ takes its minimum as claimed in Eq. (14).

A.2. Tightened formulation of the maximal additional heating $\dot{E}^c_{AC}$

The additional heating demand $\dot{E}^c_{AC}$ (see Eq. (14)) can be approximated even more accurately if each time step t is considered separately:

$$\dot{E}^c_{AC,t} = \min\left\{ \max_{k \in CC} \frac{\dot{V}^N_k}{\eta^{\min}_{AC}}\ ;\ \sum_{k \in AC} \frac{\dot{V}^N_k}{\eta_k} - \sum_{k \in AC} \frac{\dot{V}_{kt}}{\eta_k}\ ;\ \frac{\dot{E}^c_t}{\eta^{\min}_{AC}} - \sum_{k \in AC} \frac{\dot{V}_{kt}}{\eta_k} \right\}. \tag{A.7}$$

Here, the first term remains the same as in Eq. (14) (see Section 2.3). The second term, $\sum_{k \in AC} \frac{\dot{V}^N_k}{\eta_k} - \sum_{k \in AC} \frac{\dot{V}_{kt}}{\eta_k}$, reduces the maximal heating energy required by all absorption chillers by the amount of energy which is already reserved for the absorption chillers in the current operation. The third term takes into account the cooling demand on site $\dot{E}^c_t$ which limits the additional heating demand $\dot{E}^c_{AC,t}$ by $\frac{\dot{E}^c_t}{\eta^{\min}_{AC}} - \sum_{k \in AC} \frac{\dot{V}_{kt}}{\eta_k}$.

To obtain an MILP, we reformulate Eq. (A.7) by:

$$\dot{E}^c_{AC,t} \geq x_t \cdot \frac{\dot{V}^{\max}_{CC}}{\eta^{\min}_{AC}} \qquad \forall\ t \in T \tag{A.8}$$

$$\eta^{\min}_{AC} \leq \eta_k \qquad \forall\ k \in AC \tag{A.9}$$

$$\dot{V}^{\max}_{CC} \geq \dot{V}^N_k \qquad \forall\, k \in CC \tag{A.10}$$

$$\dot{E}^c_{AC,t} \geq y_t \cdot \left(\sum_{k \in AC} \frac{\dot{V}^N_k}{\eta_k} - \sum_{k \in AC} \frac{\dot{V}_{kt}}{\eta_k} \right) \qquad \forall\, t \in T \tag{A.11}$$

$$\dot{E}^c_{AC,t} \geq z_t \cdot \left(\frac{\dot{E}^c_t}{\eta^{\min}_{AC}} - \sum_{k \in AC} \frac{\dot{V}_{kt}}{\eta_k} \right) \qquad \forall\, t \in T \tag{A.12}$$

$$x_t + y_t + z_t = 1 \qquad \forall\, t \in T \tag{A.13}$$

$$x, y, z \in \{0, 1\}^t\,. \tag{A.14}$$

Again, Glover's linearization (see Glover, 1975) can be used to linearize the bilinear products with the binary variables x, y, and z.

Appendix B. Energy demands

Thermal demands of the industrial park also shown in Fig. 1 are listed in Table B.1 as well as the electricity demands. The energy demands are employed for calculations in Section 3.2.

The thermal and electricity demands of instance 7 of the time series variation employing latin-hypercube sampling with ±5 % are listed in Table B.2. The corresponding solutions are discussed in detail in Section 3.3.

Table B.1
Thermal demands (for cooling on Site A and Site B) as well as electricity demands; time step t_1 to t_5 are time-aggregated values; t_6 and t_7 represent minimal and maximal peak demands.

Demands	t_1	t_2	t_3	t_4	t_5	t_6	t_7
coolA in kW	1446	1780	2540	3137	3982	905	12262
coolB in kW	727	773	844	877	921	467	1714
heat in kW	4272	3308	2444	2333	2289	9463	1473
electricity in kW	5446	5443	5431	5419	5451	7911	7911

Table B.2
Thermal and electricity demands (for cooling on Site A and Site B); original time series varied by employing latin-hypercube sampling with ±5 %; time step t_1 to t_5 are time-aggregated values; t_6 and t_7 represent minimal and maximal peak demands.

Demands	t_1	t_2	t_3	t_4	t_5	t_6	t_7
coolA in kW	1415	1747	2430	3129	4168	879	11797
coolB in kW	694	788	802	844	947	473	1705
heat in kW	4205	3293	2404	2312	2378	9708	1461
electricity in kW	5710	5181	5300	5444	5342	7663	7657

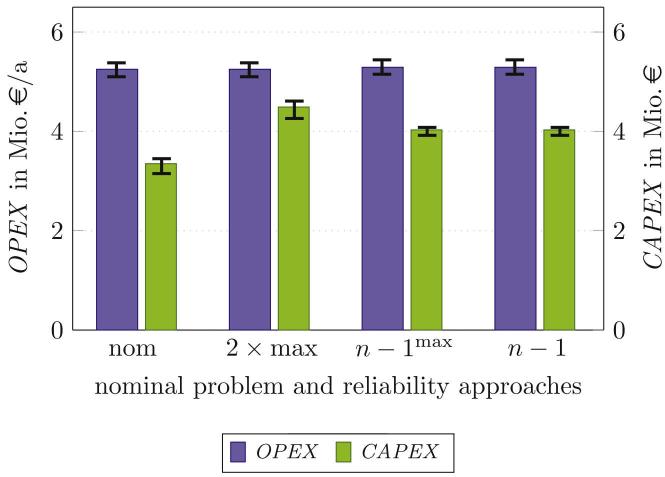

Fig. C.1. Operational costs *OPEX* and investment costs *CAPEX* for the instance leading to the median of the nominal total annualized costs; error bars show the ranges in which all solutions for the other instances lie inside.

Appendix C. Costs for solutions based on varied demand data

Fig. C.1 shows the operational costs and investment costs for instance 6. Instance 6 leads to the median of the nominal total annualized costs. Additionally, the ranges of all other instances are illustrated.

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